HARMONIC POLYNOMIAL APPROXIMATION AND INTERPOLATION

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The study of Hadamard-type inequalities for norms of harmonic functions in $\mathbf{R^n}$ leads in a natural way to the notions of Lh-functions and extremal Lh-functions [2,4,5] which play the same basic role as plurisubharmonic functions and extremal plurisubharmonic functions do in various applications of pluripotential theory to analytic functions of several complex variables. Methods of Lh-potential theory are adequate tools for studying some problems on harmonic functions such as isomorphic classification and bases for harmonic functions spaces, separate harmonicity, best polynomial approximations, harmonic polynomial interpolation, orthogonal harmonic polynomials etc. (see, e.g. [1-6]). We concentrate our attention here on two closely connected applications: best approximation and interpolation by harmonic polynomials in $\mathbf{R^n}$. The connection between the rate of convergence of those approximations and the range of harmonic extension is investigated using two versions ("inner" and "outer") of *Lh-Green functions* $h_K^{\pm}(t)$ *for a compact set* $K \subset \mathbf{R^n}$, introduced in [4,2]. In particular, we reveal, for $n \geq 3$, an irreducible gap between the necessary and sufficient conditions for domains of harmonicity providing a given rate for the best harmonic polynomial approximations or of the extremal harmonic interpolation polynomials. Notice that the case $n = 2$ was investigated earlier using the traditional potential theory (J. L. Walsh, J. H. Curtiss, J. Siciak et al), due to close connection between harmonic and analytic functions in this case.

B I B L I O G R A P H Y

1. Skiba N. and Zahariuta V. *Asymptotics of Kolmogorov diameters for some classes of harmonic functions on spheroids* / V. Zahariuta // J. Appr. Theory. 2000. V. 102. P. 175-199.

2. Skiba N. and Zahariuta V. *Bernstein-Walsh Theorems for harmonic functions in* $\mathbf{R^n}$ / N. Skiba and V. Zahariuta // Israel Mathematical Conferences Proceedings. 2001. V. 15. P. 357-382.

3.Skiba N. and Zahariuta V. *Harmonic transfinite diameter and Chebyshev constants* / N. Skiba and V. Zahariuta // Complex Analysis and Potential Theory. Proceedings of the Conference Satellite to ICM 2006. New Jersey etc.: World Scientific. 2007. P. 357-382.

4. Zahariuta V. *Spaces of harmonic functions* / V. Zahariuta // Functional Analysis / Lect. Notes in Pure and Appl. Math. NY – Basel –Hong Kong: Dekker. 1993. V. 150. P. 497-522.

5. Zahariuta V. *Inequalities for harmonic functions on spheroids and their applications* / V. Zahariuta // IUMJ. 2001. V. 50. P. 1047-1075.

6. Zahariuta V. *On harmonic polynomial interpolation* / V. Zahariuta // Complex Analysis and Dynamical Systems III. Contemporary Mathematics. 2008. V. 455. P. 439-453.