LAGRANGE'S MEAN MOTION PROBLEM AND BEYOND

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Let $P(z)$ be an exponential polynomial $\sum_{j=1}^n c_j e^{il_j z}$, $c_j \in \mathbf{C}$, $l_j \in \mathbf{R}$. The famous mean motion problem which goes back to Lagrange is the existence of the limit

$$c(y) = \lim_{\beta-\alpha\to\infty} \frac{\Delta_{\alpha<x<\beta} \arg P(x+iy)}{\beta-\alpha}$$

This problem was proved by B. Jessen and H. Tornehave (1945). They also showed the existence of the limit for every holomorphic almost periodic function $f(x+iy)$ on a strip $a<y<b$ for all $y \in (a,\, b)$ outside of some countable set. Moreover, $c_f(y) = -J_f'(y)$, where

$$J_f(y) = \lim_{S\to\infty} (2S)^{-1} \int_{-S}^{S} \log|f(x+iy)|dx$$

is the *Jessen function* of f.

Next, if the spectrum of an almost periodic function f lies in the ray $(a, +infty)$, then f extends to the upper half-plane as a holomorphic almost periodic function. Moreover,

$$-\Lambda = \lim_{y\to+\infty} \frac{\log|f(iy)|}{y} = \lim_{y\to+\infty} \frac{J_f(y)}{y} = \lim_{y\to+\infty} J_f'(y) = -\lim_{y\to+\infty} c_f(y),$$

where $\Lambda = \inf \mathrm{sp} f$. The proof is rather complicated in the case $\Lambda \notin \mathrm{sp} f$, it is the contents of Levin's Secular Constant Theorem (1941).

On the other hand, H.Bohr (1926) investigated asymptotic properties of these functions in the complex plane. He showed that in the case $\Lambda \ge 0$ the function $f(z)$ tends to a finite limit as $y \to +\infty$, if $\Lambda < 0 \,\&\, \Lambda \in \mathrm{sp} f$, then $f(z) \to \infty$ as $y \to +\infty$, and if $\Lambda < 0 \,\&\, \Lambda \notin \mathrm{sp} f$, then $f(z)$ takes every complex value on each half-plane $y > q \ge 0$.

In our talk we suggest multidimensional analogues of Lagrange's mean motion theorem and Levin's Theorem. Also, we investigate asymptotic properties of almost periodic functions in $\mathbf{R}^k$ with spectrum in a cone.